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CHANGES IN THE TYPE OF THE SHOCK-WAVE STRUCTURE IN HIGH-VELOCITY FLOWS

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Physical aspects of nonuniqueness of shock-wave structures in supersonic and hypersonic flows are considered. Thermodynamic conditions determining the dual solution domains are analyzed, and the boundaries of the transition from Mach to regular reflection are examined.

Key words: *mathematical simulation, gas dynamics, solution nonuniqueness, shock waves, Mach and regular reflection.*

Evolution of mathematical simulation methods due to high-quality performance of advanced computers made it possible to study three-dimensional high-velocity gas flows with complicated shock-wave structures. Investigation of nonuniqueness and hysteresis of numerical solutions obtained and analysis of the degree of their adequacy to real physical processes become more and more important. Nevertheless, the approach based on postulates of mechanics, which sometimes involves too many mathematical details, with an analysis of the classical gas-dynamic Euler equations closed by the equation of state of an ideal gas, still prevails in gas dynamics. The physics of the process (excitation of vibrational and rotational degrees of freedom of molecules, their dissociation, recombination, and ionization of atoms), however, is often ignored. This was more or less admissible in studying supersonic flows because the influence of physical and chemical processes was not very strong, but the effect of these processes on the gas medium with hypersonic velocities can be fairly noticeable.

The main objective of the present work is the analysis of shock-wave interaction if a dual solution is possible: both regular and Mach types of interaction of shock waves can exist under identical governing parameters of the problem. The results and conclusions of [1] are also discussed, which involved a comprehensive (experimental and numerical) study of the possibility of “artificially” changing the steady pattern of interaction of shock waves generated by a high-velocity gas flow incident onto a system of two wedges with apex half-angles β_1 and β_2 (Fig. 1), with a transition from Mach reflection of shock waves to regular reflection. For this class of problems, there is a range of parameters $[\beta^*, \beta^{**}]$, in which, under the condition

$$\beta^* \le \beta \le \beta^{**} \tag{1}$$

the laws of conservation (of mass, momentum, and energy) admit the existence of both Mach and regular reflection (Neumann paradox). Outside the range determined by Eq. (1), there can exist only regular reflection if

$$\beta < \beta^*, \tag{2}$$

or only Mach reflection is

$$\beta > \beta^{**}. \tag{3}$$

Conditions (1)–(3) are written for the symmetric case $\beta_1 = \beta_2 = \beta$ (see [2] for more details). Intensification of research associated with the development of hypersonic flying vehicles and elements of its engine stimulated the practical interest in studying seemingly purely mathematical problems, such as nonuniqueness and bifurcation of solutions of equations (in our case, gas-dynamic equations).

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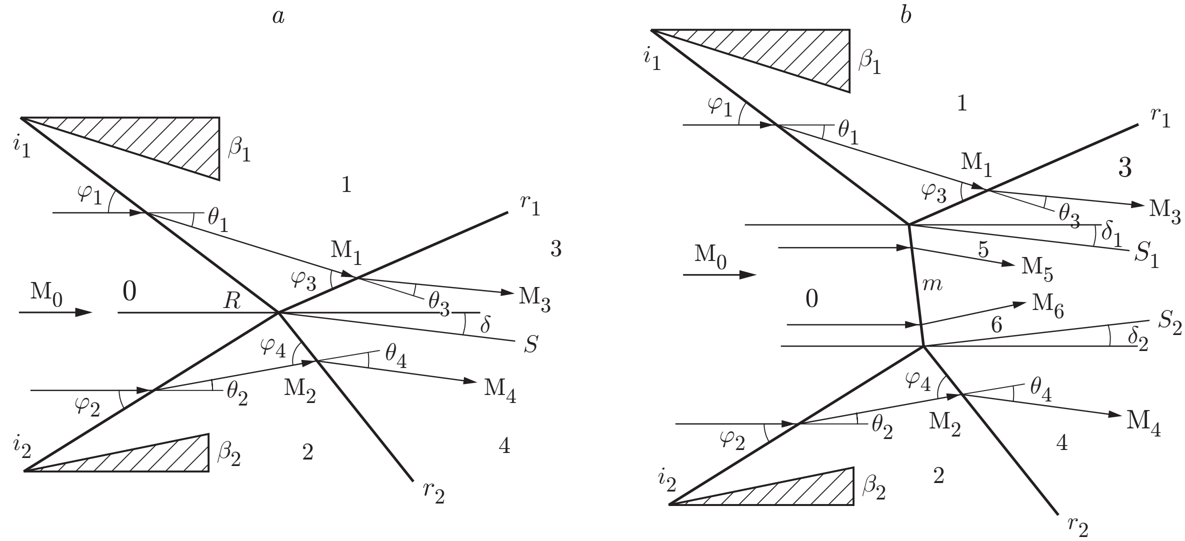

Fig. 1. Shock-wave structures generated by shock-wave interaction: (a) regular reflection; (b) Mach reflection.

For high-velocity flying vehicles, injection of the precompressed oxidizer (air) into the duct of a hypersonic air-breathing engine (scramjet) is completely determined by the flight velocity and the diffuser geometry, which have to ensure stability and predictability of scramjet operation, in addition to optimization of the flow-rate parameter. A system of oblique shock waves responsible for the flow structure in the duct is formed at the scramjet-diffuser entrance. For the scramjet to operate in the design mode, a system for correcting the inflow into the diffuser should be developed. Most of such systems are mechanical and are based on the possibility of varying the inflow angle. The idea of the "thermal correction" of the diffuser [3] seems to be promising, which implies energy input into the incoming flow upstream of the diffuser. Such a correction (as well as the mechanical correction), however, cannot guarantee complete elimination of off-design regimes, moreover, in maneuvering of the flying vehicle. One off-design regime involves the incidence of the oblique shock wave inside the diffuser and its reflection, which can cause flow separation or formation of a circulation flow. This leads to significant nonuniformity of the flow and to high thermal and force loads. Therefore, it is important to study such regimes at different flight altitudes and velocities and to predict their consequences.

Yan et al. [1] considered the possibility of changing the shock-wave interaction mode by means of heat addition to the flow. In our opinion, however, the interpretation in [1] is incorrect: this is not the transition from Mach to regular reflection but generation of new thermodynamic conditions in the gas flow under which relation (2) is valid rather than relation (1). Correspondingly, Mach reflection becomes impossible, and only regular reflection is observed.

The change in interaction modes should be understood as the solution nonuniqueness under the same thermodynamic conditions in domain (1), where the existence of both regular and Mach reflection is possible, though only one of these regimes is naturally realized.

Let us analyze the results of [1] from the viewpoint of postulates developed in [2, 4–6]. A schematic of two shock-wave configurations formed due to reflection of shock waves in steady flows is shown in Fig. 1. The regular configuration (Fig. 1a) formed by the incidence of a supersonic flow with a Mach number M_0 onto two wedges with angles β_1 and β_2 includes two oblique shock waves i_1 and i_2 formed near the wedge surface and incident inward the flow region with angles of inclination φ_1 and φ_2 (hereinafter, the angles are determined relative to the free-stream vector) and two reflected shock waves r_1 and r_2 with angles of inclination φ_3 and φ_4 intersecting at the point R. The cocurrent stream S with an angle of inclination δ is formed as the flow passes through the system of shock waves with flow-deflection angles θ_1, θ_2, θ_3, and θ_4 on the shock waves i_1, i_2, r_1, and r_2, respectively. The following relations are valid for the steady configuration: $\theta_1 = \beta_1$, $\theta_2 = \beta_2$, and $\theta_1 - \theta_3 = \theta_2 - \theta_4 = \delta$. For symmetric reflection ($\beta_1 = \beta_2$), we have $\delta = 0$. If the Mach configuration arises (Fig. 1b), the central shock wave m is formed in addition to incident and reflected shock waves; the front of the central shock wave connects two triple points of shock-wave intersection (i_1, r_1, m) and (i_2, r_2, m), and two cocurrent streams S_1 and S_2 with angles of inclination δ_1 and δ_2 are formed. In the steady configuration, we have $\theta_1 = \beta_1$, $\theta_2 = \beta_2$, $\theta_1 - \theta_3 = \delta_1$, and $\theta_2 - \theta_4 = \delta_2$. In the case of symmetry ($\beta_1 = \beta_2$), it is obvious that $\theta_1 = \theta_2$ and $\delta_1 = \delta_2 = 0$. The entire flow is divided into a number of domains with specific characteristics of the flow (which is uniform in an idealized formulation) in each domain. Domain 0 (free-stream domain) is bounded on the left by an arbitrary boundary placed in the region of the incoming supersonic flow (e.g., by a straight line connecting the wedge apices) and on the right by the fronts of the shock waves i_1 and i_2 (in addition, by the front m for Mach reflection). Domain 1 (region of the flow turned clockwise on the shock wave i_1 along the upper wedge surface) is bounded by the fronts of the shock waves i_1 and r_1 on the left and on the right, respectively. Similarly, domain 2 (region of the flow turned anticlockwise on the shock wave i_2 along the surface of the lower wedge) is bounded by the fronts of the shock waves i_2 and r_2 on the left and on the right, respectively. Domain 3 (region of the flow turned anticlockwise on the shock wave r_1) is bounded by its front and by the slip surface, which is the boundary of the cocurrent stream S (for Mach reflection, S_1). Domain 4 (region of the flow turned clockwise on the shock wave r_2) is bounded by its front and by the slip surface, which is also the boundary of the cocurrent stream S (for Mach reflection, S_2). In the case of regular reflection, domains 3 and 4 have a common boundary (are directly adjoint); in the case of Mach reflection, domains 3 and 4 are separated by domains 5 and 6 (regions of the flow behind the front of the shock wave m).

Transitions between these two types of reflection are determined by the detachment criterion and by the von Neumann criterion. These two criteria (bifurcation points) separate three domains where only Mach reflection, both Mach and regular reflection, or only regular reflection can exist. Transitions between these types of reflection with variation of parameters determining the physics of the problem, e.g., flight velocity and altitude, can be accompanied by a hysteresis. The wave structures of these types of reflection are normally considered under the assumption of unchanged physical properties of the gas flow as it passes through the entire system of shock waves, i.e., the model of an ideal polytropic gas with a constant ratio of specific heats γ in the entire flow domain is used. Such a physical model, nevertheless, should be extended to analyze real processes.

For this purpose, in the present work, we use the method of an effective ratio of specific heats (see [5–7]), which allows us to simulate the gas flow with real gas effects by varying the ratio of specific heats

$$\gamma = \gamma(p, T), \tag{4}$$

depending on local values of pressure p and temperature T. The data for xenon used as a test gas in the experiment [1] for determining the value of γ can be taken from some thermodynamic tables.

In the classical gas dynamics, the critical angles β^* and β^{**}, which are bifurcation points for the solution [boundaries of nonuniqueness of (1)], depend only on the problem geometry and two dimensionless parameters: free-stream Mach number M_0 and ratio of specific heats γ_0 constant in all domains of the flow:

$$\beta^* = \beta^*(\beta_1, \beta_2, \mathrm{M}_0, \gamma_0), \qquad \beta^{**} = \beta^{**}(\beta_1, \beta_2, \mathrm{M}_0, \gamma_0). \tag{5}$$

For hypersonic flows, where the physics of the processes in the gas medium have to be taken into account, the angles β^* and β^{**} depend on the dimensional values of physical parameters, namely, pressure and temperature, different in all flow domains:

$$\begin{gathered}\beta^* = \beta^*(\beta_1, \beta_2, p_0, T_0, p_1, T_1, p_2, T_2, p_3, T_3, p_4, T_4),\\ \beta^{**} = \beta^{**}(\beta_1, \beta_2, p_0, T_0, p_1, T_1, p_2, T_2, p_3, T_3, p_4, T_4)\end{gathered} \tag{6}$$

(p_i and T_i, where $i = 0, 1, 2, 3$, and 4 are the values in the corresponding flow domains).

If we consider only equilibrium or quasi-equilibrium processes, we can use a simpler physical model instead of (6), with allowance for (4), up to certain critical values of p and T (see more details in [5, 6] based on the classical publications [7–9]):

$$\beta^* = \beta^*(\beta_1, \beta_2, \mathrm{M}_0, \gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4), \qquad \beta^{**} = \beta^{**}(\beta_1, \beta_2, \mathrm{M}_0, \gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4). \tag{7}$$

To analyze the wave structures generated by interaction of the incident shock waves i_1 and i_2 determining the formation of the reflected shock waves r_1 and r_2 of different types (see Fig. 1), it is convenient to use the technique of shock polars. This technique allows us to avoid the complicated mathematical analysis of results of the simultaneous solution of several (in accordance with the number of interacting shock waves) nonlinear algebraic equations relating the values of parameters upstream and downstream of the front of each shock wave. Selection

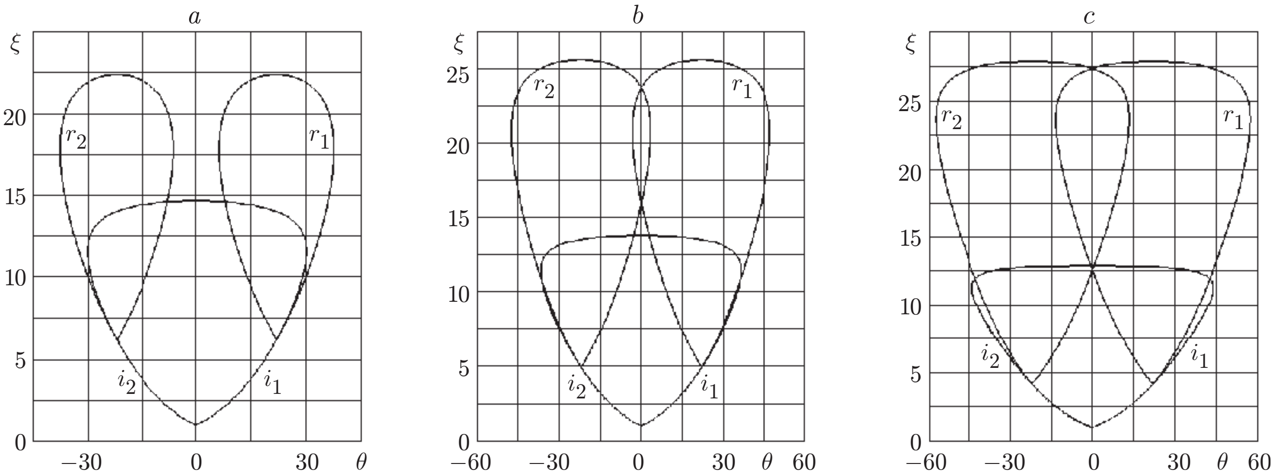

Fig. 2. Polars of two incident (i_1 and i_2) and two reflected (r_1 and r_2) shock waves for $\beta_1 = \beta_2 = 22°$, $M_0 = 3.45$, $p_0 = 11.23$ atm, $T_0 = 283$ K, and $\gamma_0 = \gamma_1 = \gamma_3 = 1.667$ (a), 1.4 (b), and 1.2 (c).

of the solutions due to their nonuniqueness is performed by an illustrative graphical method. This method makes the mere processes of obtaining the solutions and their analysis significantly clearer and more logical and the choice of the necessary solution in the case of its nonuniqueness simpler. The shock polar is understood as an expression relating the flow-deflection angle θ and the pressure ratio $\xi = p_+/p_-$ (p_+ and p_- are the pressures behind and ahead of the shock-wave front, respectively) with a parametric dependence on the Mach number M_- and effective ratios of specific heats γ_+ and γ_-:

$$f(\theta, \xi, M_-, \gamma_-, \gamma_+) = 0. \tag{8}$$

Dependence (8), which is actually called the shock polar, in the plane $(x, y) = (\theta, \xi)$ is a closed curve bounded by the values $\theta_{\min} \le \theta \le \theta_{\max}$ and $\xi_{\min} \le \xi \le \xi_{\max}$ and specularly symmetric with respect to the straight line $\theta_s = (\theta_{\min} + \theta_{\max})/2$. The specific form of Eq. (8) and a detailed analysis of the shock polar with varied γ_+, γ_-, and M_- can be found in [2, 4].

Figure 2 shows the results of numerical simulations (the iterative numerical algorithm is described in detail in [4]) of the problem with the values of the governing parameters being the same as in [1]. The study was performed with varied values of γ_0 and, because of many parameters used in the problem, under the additional condition $\gamma_3 = \gamma_1 = \gamma_0$. This condition means that the basic physical processes in the gas medium (xenon), namely, excitation of electron shells and partial ionization of atoms, proceed upstream of the fronts of the incident shock waves, and the gas properties remain unchanged in passing through the fronts of reflected (weaker) waves. In addition, because of the problem symmetry, we have $\gamma_2 = \gamma_1$ and $\gamma_4 = \gamma_3$. The effective ratio of specific heats γ reflects the physics of the process: its value equals 5/3 for a monatomic gas in the free stream, decreases because of excitation of electron shells, and increases owing to ionization. Note, an analysis of processes from this viewpoint in a number of problems of dynamics of a reacting gas [10–13] with γ reduced to 1.22 in argon and to 1.06 in alcohol vapors made it possible to explain some phenomena anomalous from the viewpoint of the classical gas dynamics, such as instability and breakdown of the bow shock wave in a hypersonic flow around a blunted body.

Figure 2 shows the polars of two incident (i_1 and i_2) and two reflected (r_1 and r_2) shock waves. Because of the symmetry, the shock waves i_1 and i_2 coincide, and the shock waves r_1 and r_2 are specularly symmetric with respect to the line $\theta = 0$. The relative position of the points of intersection of these polars determines the type of the resultant shock-wave configuration. Even if the numerical values of β^* and β^{**} in (7) are unknown, the shock-polar patterns suggest which of the shock-wave structures is formed for a certain set of parameters. Let us write these conditions, similar to (1)–(3), in the same sequence but in a different formulation:

1) if the polars r_1 and r_2 intersect outside the polar i_1, both regular and Mach reflection is possible;

2) if the polars r_1 and r_2 intersect inside the polar i_1, Mach reflection is impossible;

3) if the polars r_1 and r_2 do not intersect, regular reflection is impossible.

Let us emphasize that, if there are two points of intersection of ($r_1 \times r_2$) polars, only the lower of them (called the main polar or the weak solution) should be used. This issue was analyzed in detail in [2, 4].

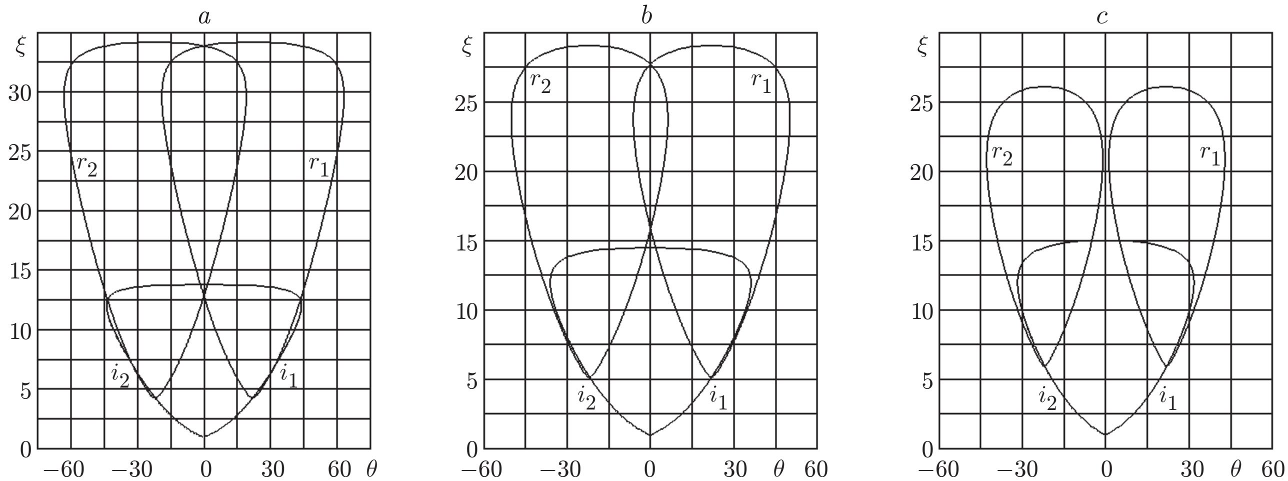

Fig. 3. Polars of two incident (i_1 and i_2) adn two reflected (r_1 and r_2) shock waves for $\beta_1 = \beta_2 = 22°$, $\mathrm{M}_0 = 3.45$, $p_0 = 11.23$ atm, $T_0 = 283$ K, and $\gamma_0 = 1.3$ (a), 1.5 (b), and 1.667 (c); $\gamma_1 = \gamma_0 - 0.05$ and $\gamma_3 = \gamma_1 - 0.05 = \gamma_0 - 0.1$.

Figure 2a illustrates the initial state of the steady flow in the experiment [1]: the structure with stable Mach reflection corresponding to Fig. 1b is formed. Then, a strictly localized "spot" of rather powerful laser radiation is inserted in a short pulse in the region ahead of the front of the shock wave i_1 in the vicinity of the apex of the upper wedge. Because of substantial changes in free-stream parameters, some part of the front of the shock wave i_1 is deformed, and this deformation gradually moves toward the center. By the time of 100 μsec, the disturbance enters the region of the central shock wave, which disappears, and a pattern of the other type (regular reflection) arises; as is argued in [1], this pattern exists for a certain time. According to [1], this is how the transition from Mach to regular reflection occurs, whereas the back transition from regular to Mach reflection is explained by the influence of perturbations in the wind-tunnel duct.

From the viewpoint of the present study, another explanation for this process is more reasonable. We have $\gamma = 1.667$ in all regions of the undisturbed xenon flow in general and, in particular, at the initial time until the heat "spot" moves to the center. According to condition (3) (see Fig. 2a), only the shock-wave structure corresponding to Mach reflection can arise. This is the experimentally observed event. After that, as the edge of the heat "spot" smeared by the flow moves to the center, the value of γ decreases and reaches the value of, e.g., 1.4 at a certain intermediate time. In this case (see Fig. 2b), the point of intersection of the $(r_1 \times r_2)$ polars lies outside the polar i_1. Thus, the existence of both Mach and regular reflection is possible in this domain of parameters. Note that the value $\gamma = 1.4$ is typical for diatomic gases, in particular, for the main components of air at moderate temperatures where only translational and rotational degrees of freedom are excited and there is no substantial activation of atomic oscillations in gas molecules.

When the disturbance reaches the center, the latter is a region of a strongly heated gas with low values of $\gamma_i < \gamma_0$ $(i = 0, 1, 2, 3, 4)$, which cannot be determined exactly because of the lack of necessary information. Most probably, the value of 1.2 is reached here. In this case (see Fig. 2c), in contrast to Fig. 1a, the point of intersection of the polars r_1 and r_2 lies inside the polar i_1. Hence, in this situation, Mach reflection is forbidden, and only regular reflection is possible. This situation is observed in the experiment. After a certain time, the heat "spot" is entrained downstream, and the initial situation is restored, in which both regular and Mach reflection is possible at first (see Fig. 2b) and then only Mach reflection is possible (Fig. 2a).

The final stages of reconstruction of the initial shock-wave structure are illustrated in more detail by a numerical experiment whose results are shown in Fig. 3. The effective ratios of specific heats are different in different flow domains and decrease with increasing domain number: $\gamma_1 = \gamma_0 - 0.05$ and $\gamma_3 = \gamma_1 - 0.05 = \gamma_0 - 0.1$. This means that physical and chemical processes (in xenon, excitation of electron shells) increasing the temperature and, correspondingly, decreasing the effective ratio of specific heats proceed in a jumplike manner on the fronts of the incident and reflected shock waves. The problem, naturally, retains its symmetry: $\gamma_2 = \gamma_1$ and $\gamma_4 = \gamma_3$. The parameter γ_0 is varied.

This dynamics of variation of γ_0 and related values of γ_1 and γ_3 corresponds to downstream "entrainment" of the heat "spot," and the "spot" itself is fairly smeared (in this approximation). As $\gamma_3 < \gamma_1 < \gamma_0$, the temperatures obey the inverse dependence: $T_3 > T_1 > T_0$, i.e., the "spot center" (region with the maximum temperature) is located more downstream than the "spot periphery." In addition, since $\gamma_0^c > \gamma_0^b > \gamma_0^a$ (the superscript indicates the numeration in the figure), we have $T_0^c < T_0^b < T_0^a$. This describes a gradual decrease in temperature (if we consider the process evolution as a–b–c in Fig. 3) not only in space but also in time. By the end of the process (Fig. 3c), the initial free-stream parameters are restored: $\gamma_0^c = \gamma_0^{t=0} = 1.667$.

Let us consider the results of this numerical experiment. The following dynamics of shock polars is observed: the polars r_1 and r_2 intersect inside the polar i (Fig. 3a), intersect outside the polar i (Fig. 3b), and do not intersect (Fig. 3c). This means that the following configurations are possible: only regular reflection, both regular and Mach reflection (dual-solution domain), and only Mach reflection. Thus, as the heat "spot" is entrained downstream, the initial configuration of Mach reflection is restored, which is the situation observed in the experiment.

Hence, the formulation of the experiment [1] can hardly be treated as organization of conditions for the transition from Mach to regular reflection in the dual-solution domain: this can be considered as generation of thermodynamic conditions for the domain of uniqueness of the other solution to exist. If it is necessary (or desirable) to convert the shock-wave structure corresponding to Mach reflection to the regular configuration, the simplest method is to reduce the flow-deflection angles β_1 and β_2 from 22° to, e.g., 10°. In this case, only the regular configuration is obviously possible (see [2, 4]).

Similar comments on the incorrect use of the term "transition" can be referred to some other papers where the type of the shock-wave configuration is changed by varying free-stream conditions. For instance, the parameter varied in the numerical experiments [14] is the density in domain 0 (see Fig. 1) in the vicinity of the plane of symmetry. Actually, this is equivalent to replacing the steady value $\mathrm{M}_0 = \mathrm{M}_s$ by the disturbed value $\mathrm{M}_0 = \mathrm{M}_d$ and, according to (5), (6), or (7), reorganization of the "solution-attraction basin," namely changing the positions of the bifurcation points β_s^* and β_s^{**} to β_d^* and β_d^{**} in the space of parameters. Correspondingly, the conditions of formation of the shock-wave structure becomes different: only Mach reflection could be formed previously (for $\mathrm{M}_0 = \mathrm{M}_s$), whereas only regular reflection becomes possible for $\mathrm{M}_0 = \mathrm{M}_d$. Note, of greatest interest is the case in the dual-solution domain where it is not clear *a priori* which type of reflection will be formed. Therefore, in our opinion, the experiment [14] cannot be classified as organization of the "transition" between two different types of interaction: one structure can exist for some values of parameters, and the other structure is possible for different parameters.

Generally speaking, it is extremely important to know which type of the shock-wave configuration will be formed not only in the theoretical aspect but also for practical applications in the development of hypersonic flying vehicles. This aspect of the problem was partly analyzed above, but it should be supplemented by one more comment. In analyzing the possibility of this or that regime of shock-wave interaction and its stability to transition, it seems extremely important to formulate a numerical experiment that would not be confined to taking into account disturbances of one parameter (the list of parameters and their combinations are infinite) but would make comprehensive (at least averaged) allowance for real physical and chemical processes in the gas (air) with significant variations in the properties of the gas medium.

The conclusion on mandatory allowance for changes in thermodynamic properties of the gas in hypersonic flows with high-pressure and high-temperature regions is illustrated in Fig. 4, which shows the ratio of specific heats c_p/c_v for air (classical ratio of specific heats $\gamma = c_p/c_v$) as a function of temperature for different pressures (the data were taken from tables in [9]). The wavy behavior of the curves is caused by physical processes consecutively proceeding with increasing temperature, such as excitation of vibrational degrees of freedom of oxygen molecules and their dissociation, excitation of vibrations in nitrogen molecules and their dissociation, and excitation of electron shells of atoms and their ionization.

According to (7) and (8), the value of γ substantially affects the entire shock-wave configuration of the flow. This effect is so important (see [2, 4] for more details) that all investigations that ignore this effect are rather limited and can hardly be used for the development of a real flying vehicle.

Finally, an important fact should be noted: Why only Mach reflection is always (at least almost always) formed in the dual solution domain in the physical (not numerical) experiment, and regular reflection is never formed? Here, we can trace a direct analogy with a simpler problem of a supersonic flow incoming onto a wedge,

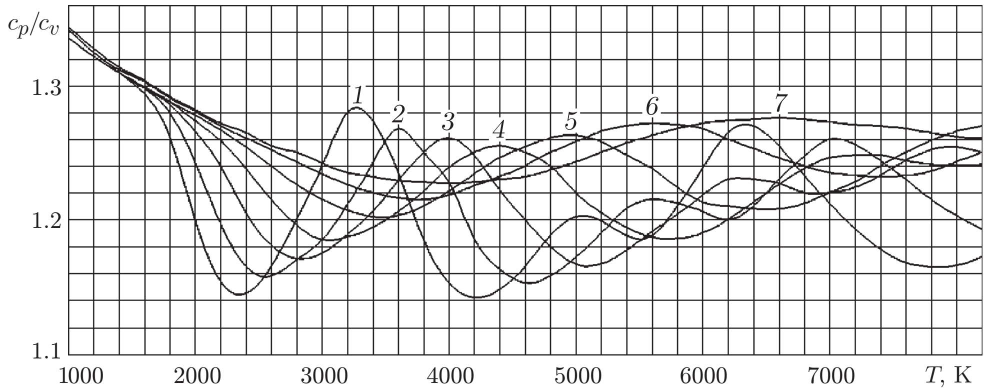

Fig. 4. Ratio c_p/c_v (for air) as a function of temperature for $P = 10^{-3}$ (1), 10^{-2} (2), 10^{-1} (3), 1 (4), 10 (5), 10^2 (6), and 10^3 atm (7).

where two solutions admitted by conservation laws are possible in a certain range of governing parameters (M_0, β): two shock waves with different intensities and different angles of the front to the incoming flow. A weaker shock wave with a smaller angle of inclination and a lower intensity is always observed in physical experiments. Moreover, numerical experiments with the strong solution used as the initial data always converted the latter into a weak solution (the author is not aware of another situation). This was explained either by some (speculative) "entropy" considerations or by instability of strong solutions.

In the first case, references are often made to the known Prigogine's thermodynamic theorem [7], which implies that the steady state corresponds to the minimum entropy production if there are external factors preventing the equilibrium state. Note, both admissible states in the dual solution domain (1) are steady. Moreover, these states are also equilibrium (in a certain approximation). Therefore, this theorem of the minimum entropy production has nothing to do with the present problem.

In the second case, stability of the flow with weak and strong types of reflection is mainly examined in the linear approximation with respect to small unsteady perturbations. Thus, it was found [15] that the problem of disturbances in the flow with a weak reflected shock wave is well-posed and the same problem with a strong shock wave is ill-posed. Stability of flows with weak shock waves and instability of flows with strong shock waves were claimed in [15] on the basis of a series of numerical experiments.

Not denying the phenomenon of instability, we can offer a more physical explanation for the preference of one of the solutions in the dual solution domain. In quantum (discrete) systems, if there are several admissible energy levels, they are filled from the lowest to the highest level: this is the fundamental principle of energy minimum. Similarly, in gas dynamics, we can consider the dual solution domain as a discrete system of energy levels in which the state with the minimum energy is reached with the maximum probability. For instance, in the unique solution domain, the main point of intersection of the $(r_1 \times r_2)$ polars is located lower than the point of intersection of the $(i_1 \times r_1)$ and $(i_2 \times r_2)$ polars; hence, a regular rather than Mach shock-wave configuration is formed. In the dual solution domain, vice versa, the main point of intersection of the $(r_1 \times r_2)$ polars is located higher than the point of intersection of the $(i_1 \times r_1)$ and $(i_2 \times r_2)$ polars; hence, a Mach rather than regular shock-wave configuration is observed. It would be of interest to perform an experiment with the transition from the basic (Mach) to the excited (regular) state in a completely identical range of parameters (1) of the gas medium.

Note, nonuniqueness of solutions of the same type arises in some subdomains of parameters determining the problem: for regular reflection, one or two points of intersection of the $(r_1 \times r_2)$ polars; for Mach reflection, one, two, three, or four points of intersection of the $(i \times r)$ polars (see [2, 4] for more details). This nonuniqueness can be considered as splitting of the basic level into a number of sublevels.

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